FIRE TEMPERATURE DISTRIBUTION IN TRAPEZOIDAL COMPOSITE FLOORS

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ABSTRACT

The paper describes a finite difference approach for calculating temperature distribution in trapezoidal composite floors exposed to fire. It is shown how the applicable differential equation is transformed to the finite difference form and the manner in which the inclined boundaries of the trapezoidal section are handled. The computer program developed is briefly described. Calculated results are compared with those from two sets of published experimental results.

INTRODUCTION

The design of structures in fire has long been based on experimentally established rules. However, fire testing of structural components is becoming increasingly expensive. With the developments in the power and accessibility of computers, there has been an increasing shift towards the use of numerical methods, validated by sufficient number of well-designed experiments. The predominant numerical method in use is the finite element method (FEM). Several commercial FEM packages of software are available which enable sufficiently accurate analysis to be carried out. These programs give rapid results, especially for linear elastic analysis. In the FEM approach, modelling of geometry does not pose a serious obstacle. For geometric and material non-linear analysis, however, these programs require considerable effort in terms of data preparation and also actual computations. An alternative approach, based on the finite differences (FD), offers a simpler formulation. However, with finite difference, modelling of geometry creates an obstacle to the development of general-purpose software. It has been found that efficient solutions can be obtained using finite differences, but the software can normally be applied to a restricted form of geometry. It is in this light that this paper presents a finite difference formulation of the problem of heat transfer in trapezoidal floors slabs, such as the one shown in Figure 1.

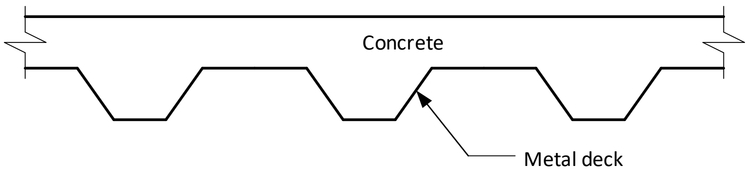

Figure 1 – Typical trapezoidal floor slab

Composite floors using profiled steel sheeting, allow rapid construction. Steel deck eliminates the need for expensive formwork. The use of metal decking provides a safe working platform during construction, provides weather protection under the metal decking and speeds up the work of other trades.

ANALYSIS FOR THE FIRE LIMIT STATE

Analysis of structural systems exposed to non-uniform heating under fire is a three-step process involving estimate of fire exposure, heat flow analysis for calculating internal temperatures, and a strength analysis to calculate deflections and eventual failure as the temperature grows. The traditional approach for the first stage is normally prescribed in the form of standard fire curves which describe the temperature of the compartment at a given time after the initiation of the fire. Since the introduction of Eurocodes, the use of natural fire curves (BS EN 1991-1-2:2002) instead of the standard fire curves has been increasingly adopted in designs since they have been shown to result in more economical, yet perfectly safe, designs. In the second stage, the classic heat flow equation is solved to determine the temperature distribution within the structure. The effects of the heat radiation and the heat convection from fire to the structure surface are accounted for via the boundary conditions. The temperatures influence the strains, which in turn, affect the stresses. Due account needs to be taken of the interaction between thermal strains and strains due to deformation of the structure. In the third stage, the equilibrium deflected shape of the structure is calculated to be in equilibrium with the applied loading, including the thermal effects. As stated above, this paper focuses on a finite difference approach for calculating the temperature distribution in trapezoidal composite floors exposed to fire.

COMPARTMENT FIRE TEMPERATURES

The intensity of fire temperatures developing in a compartment depend on the fire loading as prescribed on the basis of combustible material in the compartment expressed as mass of timber in kg/m^2. The level of fire loading is linked with the usage of the compartment. The other critical parameter is the amount of ventilation surface expressed as a fraction of the total surface of the compartment. Guidance is available in Eurocode 1 (BS EN 1991-1-2:2002) to calculate the compartment temperature with time using these parameters. An alternative, traditional, approach is to use a standardised fire curve such as ISO 834, also included in Eurocode 1 for fire, defined by the following equation:

$$T_f = T_0 + 345 \log_{10}(8t + 1) \tag{1}$$

where, T_f is the temperature of the fire in the compartment, T_0 is the ambient temperature before the fire, and t is the time (in minutes) since the commencement of the fire. The temperature unit is degree Celcius. The numerical results in the paper are for a standard fire curve, even though the computer program described later is capable of handling natural fires.

HEAT FLOW CALCULATIONS

In the context of trapezoidal composite floors, it is assumed that heat flow is two-dimensional. A common scenario is that the fire is fully developed in the volume under the floor. Even when simulating localised fire, the assumption can be justified if the two-dimensional heat flow is calculated at sufficient number of points, as in the present work.

The thermal analysis for structural fire problems can be separated into two parts. In the first part, heat transfers across the boundary from the fire into structural members by convection and radiation. The other part relates to the heat transfer within structural member by conduction. The differential equation for heat conduction in two dimensions is the well-known Fourier equation:

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) + Q = \rho c \frac{\partial T}{\partial t} \tag{2}$$

where T is the temperature, t is the time, K is the thermal conductivity, ρ is the density of the material and c is the specific heat of the material. The thermal properties of concrete and steel (thermal conductivity, specific heat, and specific mass) are temperature-dependent. The thermal conductivity decreases as a function of temperature; the specific heat increases as a function of temperature. Some standards allow computations to be performed using constant values for the thermal properties.

Equation (2) can be solved by the finite element method or by the finite difference method. The finite-difference method has been used extensively for heat conduction problems (Mills, 1994) because it is quite easy to implement. The method used here is based on the finite difference procedure given in (Lie, 1992), adapted for the geometry of the trapezoidal cross-section. Since the geometry of a typical trapezoidal metal deck floor is repetitive, only the section shown in Figure 2 is used in the analysis, using symmetry about the left axis and continuity on the right side.

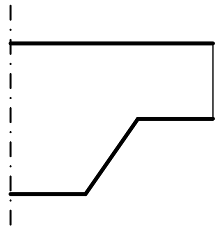

Figure 2 – Part of the cross-section selected for temperature distribution analysis

There are several finite element computer programs that solve the heat conduction equation with fire-boundary conditions such as FIRES-T3 (Iding, et al., 1977) and TASEF (Sterner & Wickstrom, 1990). The finite difference approach was adopted by Lie (Lie, 1992) for the temperature distribution calculations in reinforced concrete columns.

A paper by (Hamerlinck & Twilt, 1989) investigated the influence of specific heat of steel on the thermal behaviour of composite slabs. It was concluded that as the steel sheet is very thin, the influence of its heat capacity on the heat transfer was negligible. It has been assumed in this paper that the steel sheet at high temperatures loses its strength completely, and the floor is then reduced to a reinforced concrete slab. Thus in the modelling described below, the steel sheet is ignored.

THERMAL PROPERIES OF MATERIALS

Material properties used here are as specified in Eurocode 4 Part 1.2. The relevant properties for concrete are its density, specific heat and conductivity.

$$\rho = 2350 \qquad kg/m^2 \qquad (3)$$

$$C = 900 + 80\left(\frac{T}{100}\right) - 4\left(\frac{T}{100}\right)^2 \qquad J/kgK \qquad (4)$$

$$K = 2 - 0.24\left(\frac{T}{100}\right) + 0.012\left(\frac{T}{100}\right)^2 \qquad W/mK \qquad (5)$$

Properties of steel are not quoted here as the thin sheet of metal decking is assumed to store negligible heat energy and is neglected in the calculations.

FINITE DIFFERENCE APPROXIMATION OF HEAT CONDUCTION EQUATION

As a first step in the solution of the problem by finite differences, the domain in which the differential equation applies is divided into a mesh with gridlines parallel to the Cartesian axes. A part of the grid is shown in Figure 3. The figure shows that the spacing of horizontal and vertical gridlines (Δy and Δx, respectively) can be different in size. The figure also shows that, when the differential equation is approximated by the finite difference formulae at point (*i,j*), its influence extends over the shaded area. It is assumed that heat is conducted through the lines marked as Top, Bottom, Left and Right. In the present problem, there is no internal heat generated within the floor, hence the term Q in Equation (2) becomes zero, as do the contributions Q_{in} and Q_{out} shown in Figure 3.

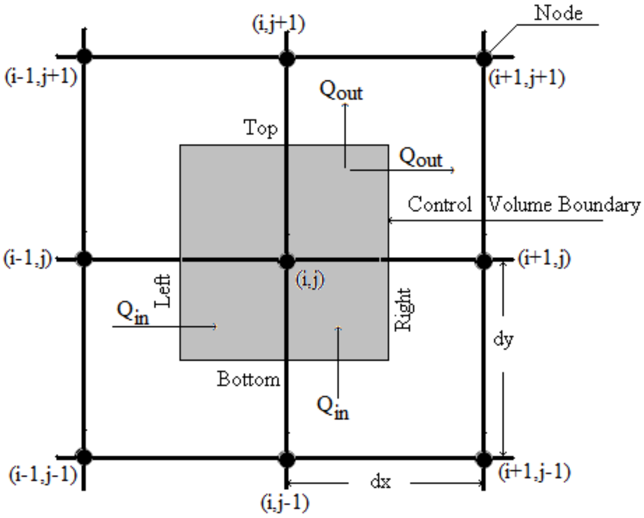

Figure 3 – Part of the grid shown, centred around the point *(i,j)* defined by vertical grid line *i* and the horizontal grid line *j*.

Without going into basics of the finite difference formulae, In the same way as Equation (2) is derived, with Q=0, the principle of conservation of energy can be applied. Thus, the rate of heat conduction into the control volume equals the rate of heat conduction out of the control volume and the rate of energy stored inside the control volume. In finite difference form, using the time step as Δt, at the temperature T^m at the mth time step, as shown below:

$$-K\left\{\frac{T_{i,j}^{m}-T_{i-1,j}^{m}}{\Delta x}\Delta y+\frac{T_{i,j}^{m}-T_{i,j-1}^{m}}{\Delta y}\Delta x\right\}=-K\left\{\frac{T_{i,+1j}^{m}-T_{i,j}^{m}}{\Delta x}\Delta y+\frac{T_{i,j+1}^{m}-T_{i,j}^{m}}{\Delta y}\Delta x\right\}+\rho c\Delta x\Delta y\frac{T_{i,j}^{m+1}-T_{i,j}^{m}}{\Delta t} \qquad (6)$$

Rearranging the above equation, the temperature at the next time step is determined as given below:

$$T_{i,j}^{m+1}=T_{i,j}^{m}+\frac{\Delta t}{\rho c\Delta x\Delta y}\left\{\frac{K(T_{i,j+1}^{m}-T_{i,j}^{m})}{\Delta y^{2}}+\frac{K(T_{+1i,j}^{m}-T_{i,j}^{m})}{\Delta y^{2}}+\frac{K(T_{i,-1j}^{m}-T_{i,j}^{m})}{\Delta x^{2}}+\frac{K(T_{i,j11}^{m}-T_{i,j}^{m})}{\Delta y^{2}}\right\} \qquad (7)$$

The above equation could have been obtained directly from equation (2). The equation also shows the power of the finite difference method. It will be noted that the complicated differential equation has been reduced to a simple algebraic equation. Also, in this particular case, the problem can be solved without iteration. A sufficient condition is that the time step has to be less than a critical value given by:

$$\Delta t<\frac{\rho c\{\min(\Delta x,\Delta y)\}^{2}}{2K} \qquad (8)$$

The significance of the sufficient condition is that it is not a necessary condition, so minor variation from the critical value does not jeopardise convergence.

Where the material properties change with temperature, the applicable conductivity parameters used, for determining the flux towards the left, right, top or bottom, are the harmonic mean (rather than the mean) of the value at point (*i,j*) and the neighbouring point depending upon the direction being considered. This is based on the proposal by (Patankar, 1980)

Thus, for the left, right, top and bottom faces of the control volume in Figure 3,

$$K_{left} = \frac{2K_{i,j}\ K_{i-1,j}}{K_{i,j} + K_{i-1,j}},\ K_{right} = \frac{2K_{i,j}\ K_{i+1,j}}{K_{i,j} + K_{i+1,j}},\ K_{bottom} = \frac{2K_{i,j}\ K_{i,j-1}}{K_{i,j} + K_{i,j-1}} \text{ and } K_{top} = \frac{2K_{i,j}\ K_{i,j+1}}{K_{i,j} + K_{i,j+1}} \tag{9}$$

Boundary Conditions

At the boundaries of the floor, both exposed to fire and ambient, emissivity of the surfaces has to be considered. Referring to Figure 4, which shows the lower horizontal boundary of the floor.

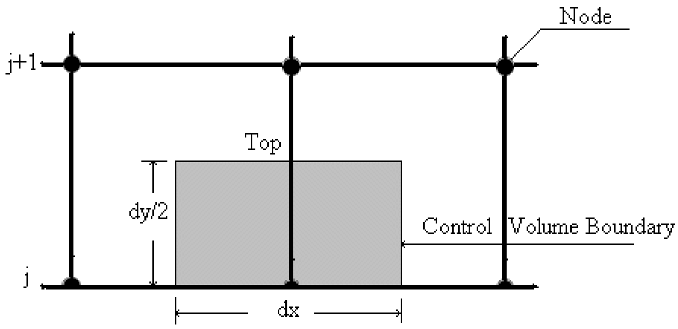

Figure 4 – Control Volume for a lower boundary

The control volume is Δx ($\Delta y/2$) shown as the shaded area in Figure 4. When writing the energy balance condition at point (*i,j*), the heat transfer from the boundary into the volume is given by:

$$Q_{i,j}^{m} = \sigma\, \varepsilon_f\ \varepsilon_c \{(T_f^m + 273)^4 - (T_{i,j}^m + 273)^4\} \tag{10}$$

Where, $Q_{i,j}^{m}$ is the heat flow in the y direction at point (*i,j*) at time *m*, σ is the Stefan-Boltzmann Constant, ε_f is the emissivity of the fire and ε_c is the emissivity of concrete. Rearranging the control volume energy balance equation gives the following equation for temperature at the next time step:

$$T_{i,j}^{m+1} = T_{i,j}^{m} + \frac{2\Delta t}{\rho c \Delta x \Delta y}\left\{\frac{K(T_{i,j+1}^{m} - T_{i,j}^{m})}{\Delta y}\Delta x + \frac{K(T_{i+1,j}^{m} - T_{i,j}^{m})}{\Delta x}\frac{\Delta y}{2} + \frac{K(T_{i,j-1}^{m} - T_{i,j}^{m})}{\Delta x}\frac{\Delta y}{2}\right\} + Q_{i,j}^{m}\Delta x \tag{11}$$

A similar equation can be written for the upper horizontal boundary.

The equation for an inclined boundary is obtained using the same procedure of writing the energy balance equation. Consider the control volume shown in Figure 5. An assumption is made that a node always exists at a boundary. This can be easily achieved by choosing Δx and Δy to match the slope of the inclined boundary. The equation for the temperature at the next time step is given by:

$$T_{i,j}^{m+1} = T_{i,j}^{m} + \frac{4\Delta t}{\rho c \Delta x \Delta y}\left\{\frac{K(T_{i,j+1}^{m} - T_{i,j}^{m})}{\Delta y}\frac{\Delta x}{2} + \frac{K(T_{i+1,j}^{m} - T_{i,j}^{m})}{\Delta x}\frac{\Delta y}{2}\right\} + Q_{i,j}^{m}\Delta x \tag{12}$$

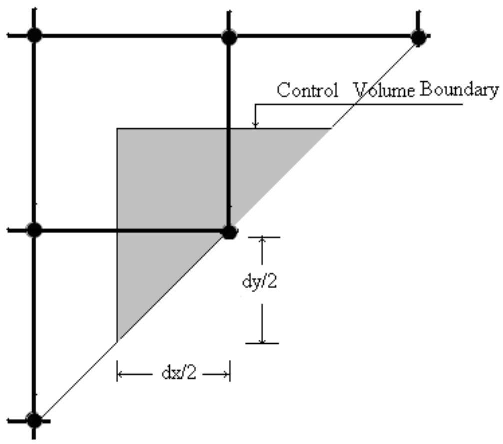

Figure 5 - Boundary control volume for an inclined node

COMPUTER IMPLEMENTATION

The equations developed above are sufficient to carry out the computations for the response to fire of composite floors exposed to fire. The flow chart of the computations is shown in Figure 6. The calculations start at the boundary nodes at the lowest level, sweeping left to right and then progressing to the higher levels one by one.

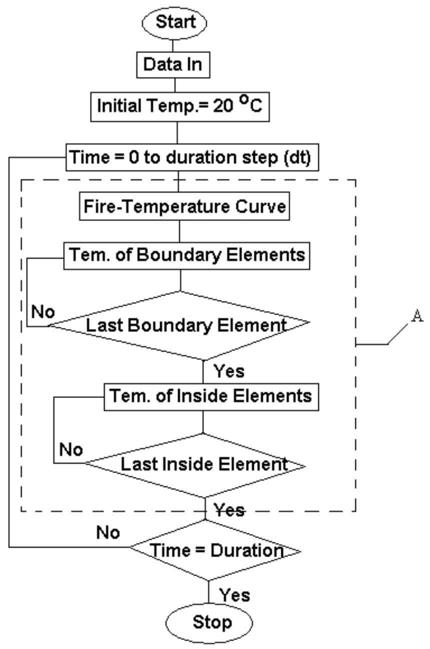

Figure 6 – Flow chart of the computations

A computer program, labelled COMFLEF has been written in Visual Basic to carry out the calculations. The program has a user-friendly interface and requires a small amount of input data to define the problem. Some key features are described below.

Geometry
The cross-section is divided into 5 zones, 4 of which are rectangular and 1 triangular, as shown in Figure 7. Only 3 horizontal and two vertical dimensions are sufficient to define the cross section. Corresponding to the 5 dimensions, the 5 values of the number of gridlines define the complete mesh. The horizontal and vertical grid equidistant spacings are deduced. It may be added that the heat

balance equations are suitably adjusted at nodes where the grid spacing at the left and right or above and below are unequal. The node numbers are generated going from the bottom to the top and then from the left to the right. As stated earlier, the metal deck is ignored in the calculations as if the concrete surface was directly exposed to the fire.

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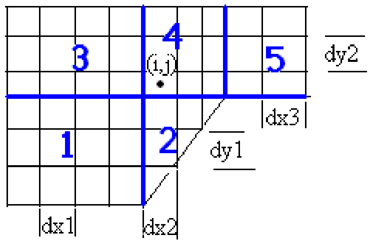

Figure 7 - Cross-section divided into 4 rectangular and 1 triangular zones

Boundary Conditions

It is assumed that the bottom surfaces are exposed to fully developed fire. The top surface is exposed to ambient temperature. The left and right side are assumed to be lines of symmetry.

Material Thermal Properties

The program has built-in material properties for concrete (and steel, although not required in this application) as specified in (BS EN 1994-1-2, 2002).

The method described above is suitable for using the temperature distributions for structural strength calculations for composite floors exposed to fire.

VALIDATION

The method described above has been validated against experimental fire tests carried out by (Hamerlinck & Twilt, 1989) and by (Abdel-Halim, et al., 1999). Comparison is made between experimental results for temperature distribution within the floor slab with results from the program COMFLEF.

The trapezoidal metal deck specimens had dimensions of 1600×700 with a total depth of 143 mm. The concrete used was normal weight concrete of grade B25 (C25 in Eurocode nomenclature). The slabs were subjected to a standard fire. Figure 8 shows the location of points where the temperatures were measured in the experiment.

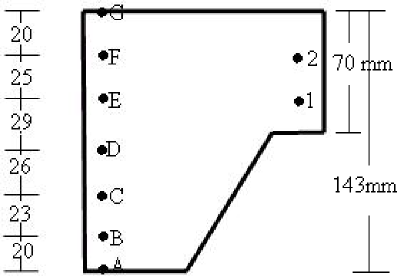

Figure 8 – Locations of points where temperatures were measured (Abdel-Halim, et al., 1999)

The distribution of temperatures down the depth is compared in Figure 9 for fire at 120 min. It will be seen that the calculated temperatures are fairly accurate

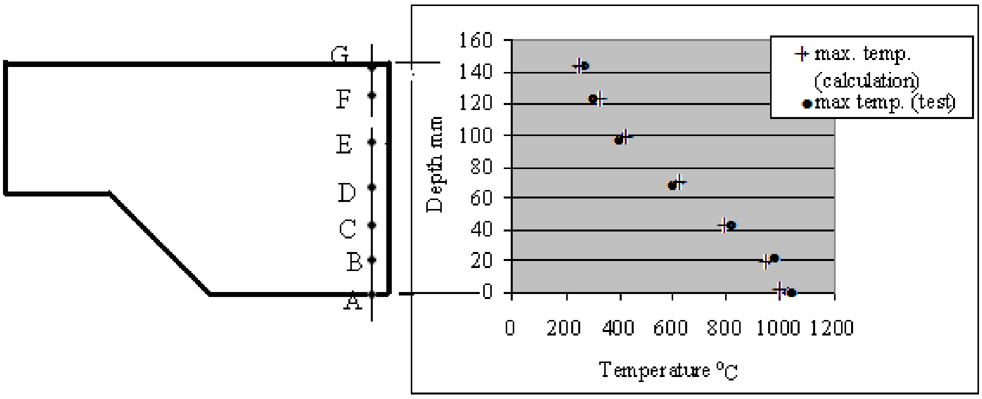

Figure 9 – Comparison of computed temperatures with measured values at 120min of fire

The cross-section and the location of selected points for measuring concrete temperatures in the tests by Abdel-Halim, Hakmi and O'Leary are shown in Figure 10 for their Sample 1, in which a light steel mesh was provided for fire resistance and to control cracking.

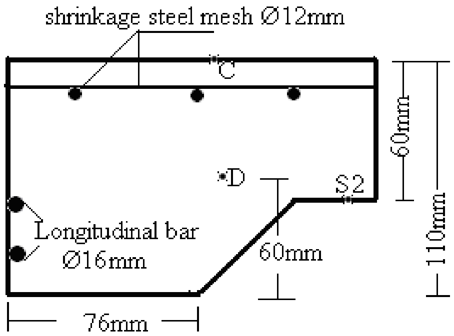

Figure 10 - Cross-section of Test Sample 1 in (Abdel-Halim, et al., 1999)

Figure 11 shows plots of the development of temperatures at various locations against time. It will be seen that the computed results follow the measured values at most locations. In all cases, the experimental results show a lag, and this is attributed to moisture in the concrete which was not taken into account in the present analysis. Clearly, the computed results are conservative.

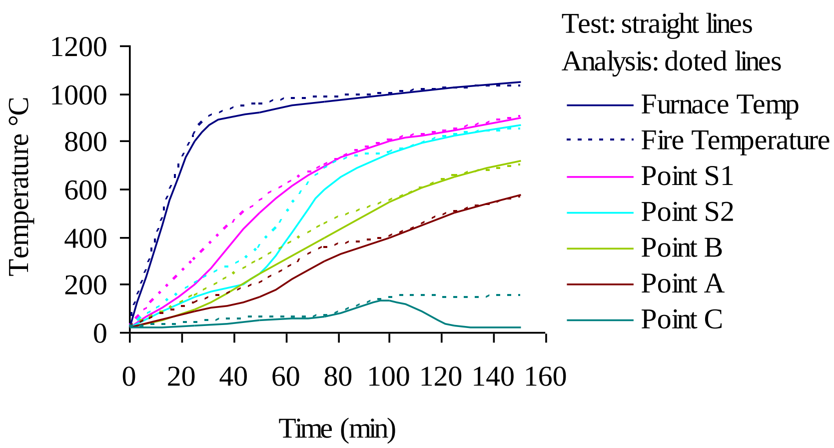

Figure 11 – Comparison between calculated and measured growth of temperatures at selected points for Sample 1 in (Abdel-Halim, et al., 1999)

CONCLUSION

A method of analysis of calculating temperature distributions in composite floors exposed to fire has been presented. The method uses the finite difference approximation to solve the Fourier equation of heat conduction. An effective solution has been obtained for dealing with inclined boundaries in the context of gridlines parallel to Cartesian axes. A computer program COMFLEF based on the method has been described in outline

The results from the computations based on the finite difference approach have been validated against experimental results from two previous publications. Excellent correlation was obtained for the temperature distribution within the cross-sections as well as for growth of temperatures at several points in the cross-section for up to 120min fire duration.

An indication has been given for using the temperature distributions calculated from the described method for mechanical response of composite floors.

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