Load-Bearing Capacity Area as an Interactive Analysis Tool in SCAD Office

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ABSTRACT

Load-bearing capacity region for structural sections and joints in terms of design codes has been considered. The main attention has been paid on important property of the region is its convexity. The paper presents a case when internal forces with not ultimate design values can be unfavorable for a non-convex load-bearing capacity region of a cross-section.
Algorithm for automatic generation of load-bearing capacity regions for structural sections, connections and joints has been presented and implemented in SCAD Office software package. The program also enables to show the position of points corresponding to the specified internal forces and to plot a convex shell on the basis of these points thus bounding the part of the load-bearing capacity region, which corresponds to any linear combination of design internal forces in the considered cross-section or structural joint. Construction of load-bearing capacity regions of a section or joint combined with a convex shell of specified internal forces is flexile tool for analysis of load conditions.
Load-bearing capacity region for cross-sections is also a mean for critical analysis of code-based requirements and regulations used for verifications of cross-sections. Unexpected properties of the design often occur here, which are caused by some simplifications and inconsistencies with the code requirements. In particular, these effects often arise due to some kinds of strict logical transitions from one design situation to another (e.g., depending on the sign of a force). This leads to an abrupt change of requirements, which does not correspond to the physical nature of the phenomenon, which is usually associated with continuous changes. In EuroCode it often deals with the change of the section class (for example, the transition from Type 3 to Type 4), which occurs abruptly at the change of the load combinations.
Finally, load-bearing capacity regions has been also used in process of software development as an instrument for critical analysis of traditional approaches to design and calculation of steel structural joints.

INTRODUCTION

Many modern software can generate interaction surfaces for various pairs (triples) of internal forces in a bar element section (see, for example, (Strelets-Streletsky and Vodopyanov 2009), (Abdelhamid Charif 2009), (Charalampakis and Koumousis 2008), (Fafitis 2001), (Rodriguez-Gutierrez and Dario Aristizabal-Ochoa 1999), (Sulimowski and Klowan 1980)). A maximum allowable in terms of strength combination of internal forces acts on this surface. Most often it is a triple of forces (M_y, M_z, N) acting in a cross-section of a reinforced concrete element. When it comes to the criteria that define the maximum allowable combination, some software are oriented toward a certain strength condition (for example, the classic Mises condition), while other programs link the analysis to the strength requirements given in the codes.
However, in both cases, the demonstration of interaction surfaces is performed only to provide a geometric illustration of the phenomenon and does not imply any active user operations.
SCAD Office, on the contrary, actively uses the generation of the load-bearing capacity areas, which differ in that they take into account the full set of code requirements (strength, general and local stability, crack resistance, etc.) to a certain structure for which the area is generated, and they also enable to use the interactive mode of their analysis. Moreover, the load-bearing capacity areas can be generated not only for the cross-sections of bar elements, but also for the joints of structural members.

LOAD-BEARING CAPACITY AREA OF SECTIONS

Design code requirements (strength conditions, general and local stability, limit slenderness, etc.) to a certain design section of a structure can be written in the form of a certain system of inequalities, each of which depends functionally on the values of internal forces $\vec{S} = \{S_1, S_2, ..., S_n\}$ that can arise in the considered section from the action of the design combinations of loadings:

$$\mathbf{\Phi}\left(\vec{S}\right) \leq 1;$$

or

$$\begin{cases} f_1\left(S_1, S_2, ..., S_n\right) \leq 1; \\ f_2\left(S_1, S_2, ..., S_n\right) \leq 1; \\ \quad ... \\ f_j\left(S_1, S_2, ..., S_n\right) \leq 1; \\ \quad ... \\ f_m\left(S_1, S_2, ..., S_n\right) \leq 1; \end{cases}$$

where n is the total number of possible internal forces in the section; m is the number of inequalities that describe the design code requirements. The value of the left-hand side of the inequality $\xi_j = f_j\left(S_1, S_2, ..., S_n\right)$ will be called *the utilization factor of restrictions*.

Each code requirement $f_j\left(\vec{S}\right) \leq 1$ defines a certain area in the *n*-dimensional space of internal forces, and the intersection of all standard inequalities $\mathbf{\Phi}\left(\vec{S}\right) \leq 1$ forms the load-bearing capacity area of the section $\mathbf{\Omega}_{\mathrm{S}}$ in terms of the considered codes (see Fig. 1). The maximum utilization factor of restrictions for each point of the load-bearing capacity area of the section is $\xi_{\max} = \max\left\{\xi_j \mid j = \overline{1, m}\right\} \leq 1$.

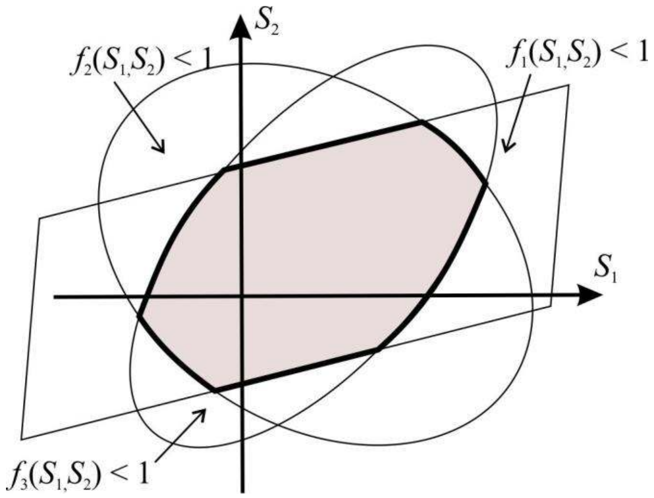

Figure 1. Formation of the load-bearing capacity area of a section in the two-dimensional space of internal forces (example)

Thus, *a load-bearing capacity area of a section* $\mathbf{\Omega}_{\mathrm{S}}$ is a certain area in the *n*-dimensional space of internal forces, all points of which correspond to a combination of these internal forces, under the action of which the cross-section satisfies *all the requirements of the considered design codes*. Unlike the classic interaction curves, the boundary of the load-bearing capacity area of the sections is not only the strength requirements of the section, but the entire set of limitations governed by the requirements given in the design codes (including the conditions of general and local stability, limit slenderness, crack resistance requirements etc.).

PROPERTIES OF THE LOAD-BEARING CAPACITY AREA OF A SECTION

One of the most important properties of the load-bearing capacity area is its convexity. It should be noted that it is the convexity of the load-bearing capacity area of the cross-section that gives us the right to limit ourselves in the linear calculation to the checks of this section for the action of only those combinations of internal forces in the cross-section for which the extreme (minimum or maximum) values are characteristic. The positive result of such checks automatically means that all other conceivable combinations of loads will be acceptable. This statement follows from the very definition of the concept of “convexity of an area”. One of its definitions says that an area is convex if and only if for an arbitrary pair of points *A* and *B* belonging to it all points belonging to the segment *AB* belong to this area as well.

The absence of the convexity property of the load-bearing capacity area of the considered section can lead to many unpleasant consequences related to the fact that, traditionally, evaluating unfavorable combinations of internal forces, engineers either do not consider some actions at all (in the case when they have a unloading effect) or take them fully into account. This rule is entirely valid for a convex load-bearing capacity area, while for a non-convex area a combination with intermediate (not extreme) values of internal forces can turn out to be an unfavorable one. Let us illustrate this with an example. Let consider a section for which a non-convex load-bearing capacity area is characteristic (Fig. 2), under the action of two independent loadings $\mathbf{P}$ and $\mathbf{\Theta}$. The point *P* in Fig. 2 corresponds to a pair of internal forces arising in the considered section under the action of the load $\mathbf{P}$, and the point *Q* – from the action of the load $\mathbf{\Theta}$. It is not difficult to see that the standard requirements are met when the section is checked for the action of each of the loads $\mathbf{P}$ and $\mathbf{\Theta}$ separately, and also when checking the section for the action of their total sum ($\mathbf{P} + \mathbf{\Theta}$), since the points *P, Q*, as well as the point *P* + *Q* belong to the load-bearing capacity area of the section. However, under the action of the “incomplete” linear combination of loadings ($\mathbf{P} + \lambda\mathbf{\Theta}$), when $\lambda < 1$, the requirements of the codes are violated, since a pair of internal forces appears in the considered section. This pair of forces is shown in Fig.2 as a point $P + \lambda Q$, which does not belong to the load-bearing capacity area.

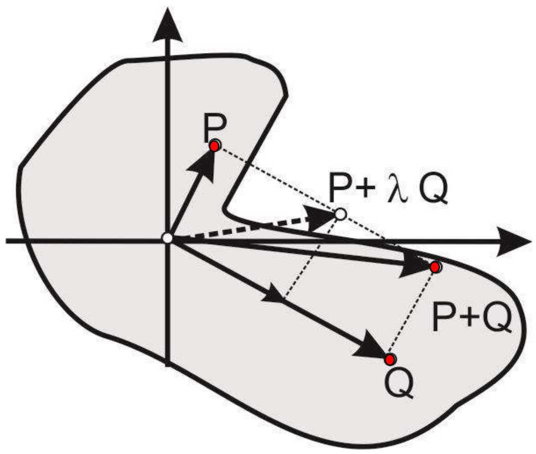

Figure 2. Checking the section for a combination of loads with “non-extreme” values

There is a firm belief in the engineering environment that the load-bearing capacity area of a section is convex. This consideration is based on the following theories:

- according to the Drucker's postulate the boundary yield surface is convex in the space of internal forces for an ideal elastically plastic system (Kachanov 1969, p. 366);
- according to the Papkovich theorem the stability area is convex in the space of loads on the system (Papkovich 1941, p. 85).

Although these theories are obviously true, they, unfortunately, do not correspond to the properties of the above-mentioned load-bearing capacity area of the section $\mathbf{\Omega}_S$, which is entirely defined by the codes. The thing is that neither the classical theory of plasticity, for which the Drucker’s postulate is valid, nor the bifurcation (Euler’s) theory of equilibrium stability, for which the Papkovich theorem is proved, is used in the design codes. Moreover, when generating the load-bearing capacity area $\mathbf{\Omega}_S$, in addition to the strength and stability conditions regulated by the design codes, it is also necessary to consider other code requirements (conditions of limit slenderness, crack resistance, etc.), which leads us far beyond the validity of the above-mentioned theories.

It should be noted that the absence of the convexity property of the load-bearing capacity area of a section is often observed in the geometrically nonlinear stability analysis. An example illustrating this fact is given below (Perelmuter and Slivker 2010, pp. 73-76).

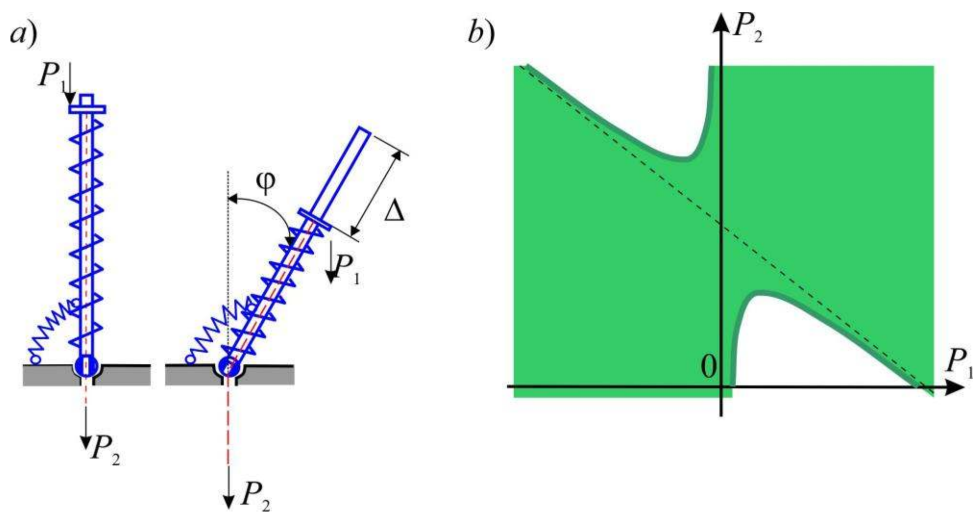

Figure 3. Example of a system with two degrees of freedom (Perelmuter and Slivker 2010, pp. 73-76)

Fig. 3, *a* shows a rigid bar elastically clamped in its initial cross-section by a spring with the rotational stiffness γ. A friction-free spring of stiffness *c* slides along the bar. The following two forces are applied directly to this spring: a vertical one P_1 and a force P_2 directed toward the bearing hinge. The mechanical realization of the force P_2 can be imagined as the tension of a rope that runs over a pulley at the bearing hinge.
The stability area shown in Fig. 3, *b*, is not convex (the reason for this is a nonlinear formulation of the problem), although the origin is within the stability area. However, it should be noted that this area does not appear to be starlike with respect to the null point. This means that there are paths of proportional loading (motion along a line starting in the null point), which first leave the stability area and then enter it again. The stability areas of ordinary designs are almost always starlike, and even if they are not, the use of the "secondary strength" has no practical value. Therefore, it is postulated in the SCAD Office analysis that the load-bearing capacity areas are starlike, and the "secondary" load-bearing capacity areas (even under the assumption of their existence) are not sought.

COMPUTER-AIDED GENERATION OF THE LOAD-BEARING CAPACITY AREA OF A SECTION

ARBAT, **KRISTALL** and **DECOR** enable to generate the load-bearing capacity areas for sections of load-bearing bar elements of reinforced concrete, steel and timber structures, respectively.
Sections of bar elements, where six internal forces (longitudinal force, bending moments with respect to the two principal axes of inertia of the section, corresponding transverse forces, and torque) can appear under the load, have a load-bearing capacity area in the form of a six-dimensional geometric object which is difficult not only to analyze, but even to represent. The best way to display the load-bearing capacity area of sections is by performing its orthogonal projection onto a certain plane (pair) of internal forces. The generation of a two-dimensional projection of the load-bearing capacity area of a section is performed according to the algorithm given below.
The user selects a pair of internal forces (for example, a pair "longitudinal force *N* – bending moment M_y"), in the coordinate system of which an orthogonal projection of the load-bearing area will be generated. The remaining internal forces in the section (M_z, Q_y, Q_z, M_x) are fixed at a certain level (they are specified by the user or take zero values). At a certain fixed value of the ratio $e = M_y/N$ (in fact, on the ray *e*), the point most distant from the origin is sought, where all the requirements of the codes are still satisfied. For such a point, a certain inequality from the system $\mathbf{\Phi}\left(\vec{S}\right) \le 1$ takes a limit value $f_j\left(\vec{S}\right) = 1$, therefore it belongs to the boundary of the two-dimensional orthogonal projection of the load-bearing capacity area of the section.

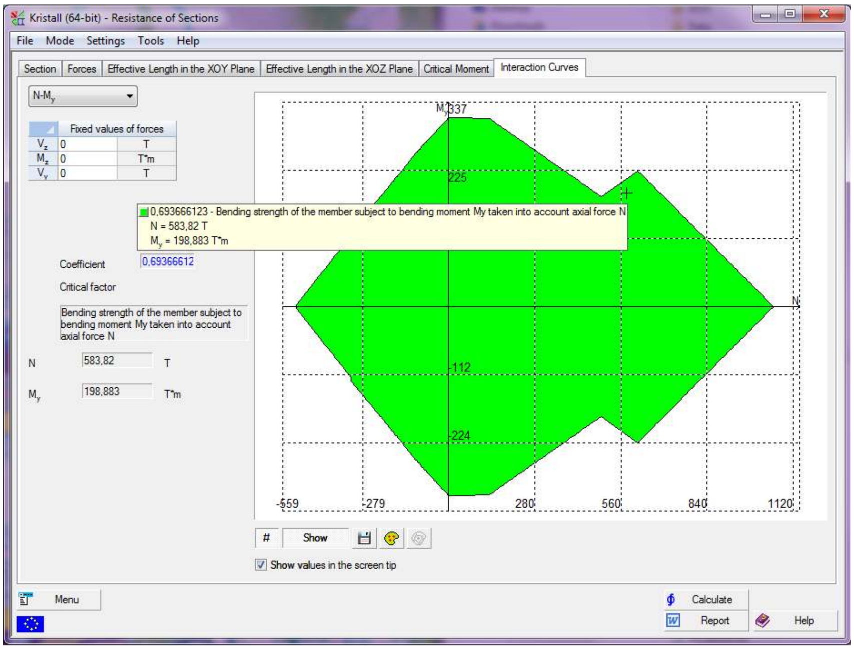

Figure 4. Interactive mode of exploring the load-bearing capacity area of the section

The entire boundary of the load-bearing capacity area is created by scanning the rays *e*, which change their value no less than every degree. In this way, two-dimensional orthogonal projections of the load-bearing capacity area are generated for any pair of internal forces selected by the user (M_y – M_z, M_y – Q_z etc.)
This method of generating a load-bearing capacity area assumes that the area is starlike, which is a hypothesis (see above), on the one hand, and a restriction of the software implementation, on the other hand.
Moreover, the generated area is an interactive tool for communicating with the user. Using your cursor, you can examine a two-dimensional projection of the area. A certain set of internal forces corresponds to each position of the cursor. Their values are displayed in the respective fields. Depending on the change in the position of the cursor (changes in the respective pair of internal forces), the maximum value of the utilization factor of standard restrictions-inequalities $\xi_{\max}$ corresponding to these forces is output, as well as the type of the inequality for which it is calculated (Fig. 4). Clicking the right mouse button on the area enables to see the entire list of performed checks and the values of utilization factors of restrictions $\xi_j \forall j = \overline{1,m}$ for the set of internal forces that corresponds to the cursor position on the generated load-bearing capacity area of the section.

LOAD-BEARING CAPACITY AREA OF SECTIONS AS A TOOL FOR THE ANALYSIS OF CODES

Several thousand calculations are performed during the computer-aided generation of the load-bearing capacity area of a section, which is apparently the largest check of the considered section. Moreover, the shape of the load-bearing capacity area of the section as well as the character of its boundaries in many cases enables to perform a more detailed analysis of the requirements of the codes than it can be done in other ways. The analysis of the boundaries of the area enables to check the consistency and completeness of the standard requirements. In this case, it is easy to identify the inconsistency of certain provisions of the codes, in particular the non-smoothness of the transition between the approximations used.
For example, let's consider the design code for steel structures SP 16.13330.2011 (SP 16.13330.2011 2011). We will generate the load-bearing capacity area for a cross-section in the form of a symmetric welded I-beam with a 400×10 mm web and 200×10 mm flanges made of steel with the design strength R_y = 2050 kg/cm^2. The effective length of the bar in both principal planes of inertia is 600 cm, the service factor and the importance factor are taken as γ_c = 1,0 and γ_n = 1,0. The load-bearing capacity area $\mathbf{\Omega}_{\text{SNiP}}$ of this section in accordance with the codes SP 16.13330.2011 is shown in Fig. 5.

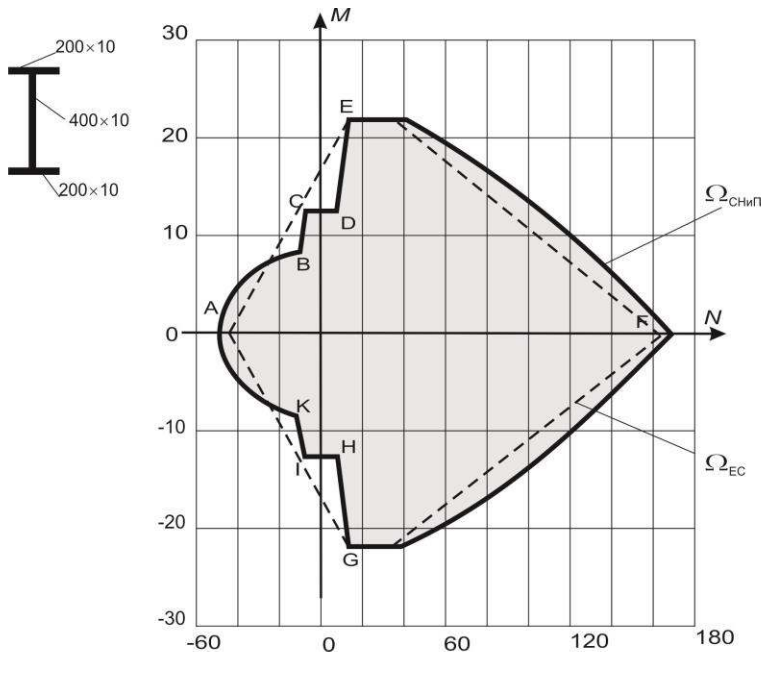

Figure 5. Load-bearing capacity area of a steel section: $\mathbf{\Omega}_{\text{SNiP}}$ – according to SP 16.13330.2011 (SP 16.13330.2011 2011); $\mathbf{\Omega}_{\text{EC}}$ – according to EN 1993-1-1:2005 (EN 1993-1-1:2005 2006)

The boundary of the load-bearing capacity area Ω_{SNiP} on the *DEFGH* section is defined by the strength condition under the combined action of tension and bending, on the *CD* and *IH* sections – by the condition of stability of in-plane bending, and on the *IKABC* section – by the condition of stability out of the bending moment plane.
The non-convexity of the boundary of the load-bearing capacity area Ω_{SNiP} on the *IKABC* section is related to the change in the type of dependence of the coefficient *c* on the value of the relative eccentricity *m*. This coefficient is included in the condition for checking out of plane buckling of a bar under bending and compression. It should be noted that the non-convexity of the *IKABC* section of the load-bearing capacity area Ω_{SNiP} does not appear when the element has small out-of-plane slenderness, for such design cases the condition of stability out of the bending moment plane is not determinative.
The configuration of the *CDE* and *IHG* sections of the load-bearing capacity area Ω_{SNiP} is determined by the codes specifying that the stability of in-plane bending of a bar should be checked only at the values of the relative eccentricity $m_x > 20$, when the stability check of such a bar has to be performed as for a flexural member. Sections of the boundary *DC* and *JK* of the load-bearing capacity area Ω_{SNiP} correspond to these values (Fig. 5).
The dashed line in Fig. 5 shows the load-bearing capacity area Ω_{EC} of a cross-section calculated according to the requirements of EN 1993-1-1: 2005 (EN 1993-1-1:2005 2006). The load-bearing capacity area of this section is convex, because the section operates within the limits of elastic deformation of steel.

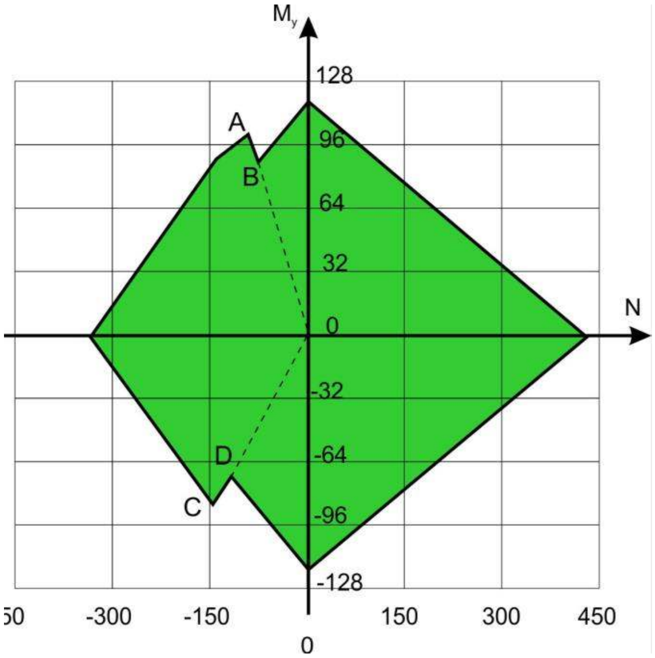

Figure 6. Load-bearing capacity area of a steel section according to EN 1993-1-1

Fig. 6 shows the load-bearing capacity area of a steel I-beam with a 800 × 10 mm web and 360 × 20 mm and 240 × 20 mm flanges generated in accordance with the requirements of EN 1993-1-1: 2005. Here the non-convexity of the load-bearing capacity area of the section is related to the requirements of EN 1993-1-1: 2005, concerning the classification of sections. In the stress state corresponding to the appearance of compressive stresses in one of the flanges, the section ceases to be classified as a section of the 2nd class (plastic deformations of steel, calculation using the plastic moment of resistance) and passes into the third class of sections (elastic deformations of steel, calculation using the elastic moment of resistance). The jumps AB and CD correspond to these transitions. A similar jump-like change in the boundary of the load-bearing capacity area can occur at the transition of the section from the 3rd class to the 4th one.

Another instructive example is given in Fig. 7, which shows the load-bearing capacity area of a reinforced concrete section calculated according to the deformation model without taking into account (variant *a*) and taking into account (variant *b*) the random eccentricity. The eccentricity is obviously taken into account "unphysically" in the variant given in the codes, since there is no physical model that would violate the smoothness of the boundary of the load-bearing capacity area. Indeed, the recommendations of the codes that for a statically indeterminate system the value of the eccentricity of the longitudinal force is taken equal to the value *e* obtained from the static analysis but not less than a certain fixed value of the random eccentricity e_0, does not in any way explain the fact that the random eccentricity (this factor is quite objective) disappears somewhere when $e > e_0$. And for a statically determinate system, the value $(e + e_0)$ is introduced into the calculation.
The number of examples could be increased, but the ones given here indicate a very real situation when the load-bearing capacity area acquires a strange configuration. As the analysis shows, in many cases there are some inconsistencies in the formulation of requirements for the elements of load-bearing structures, due most likely to insufficient adjustment of the formulations themselves.

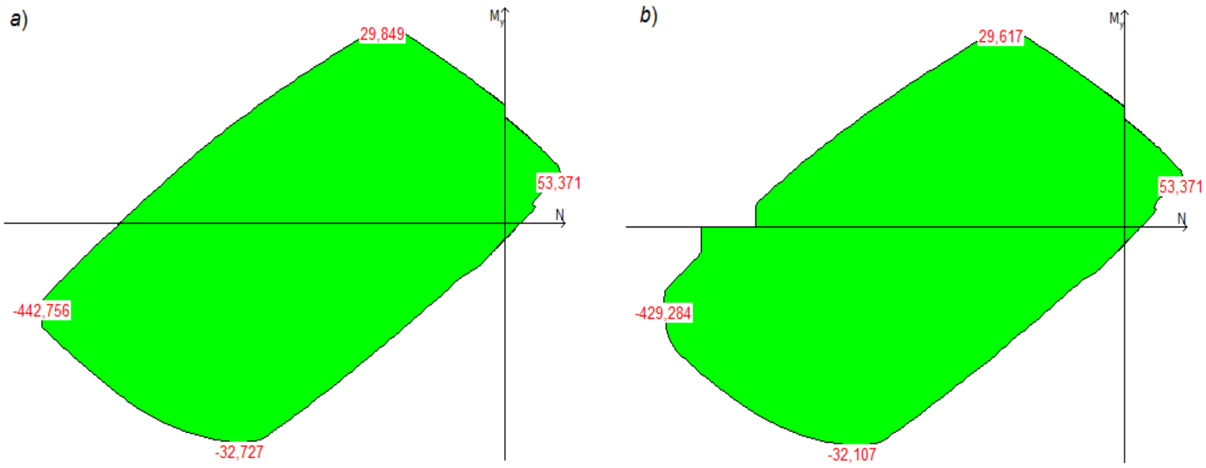

Figure 7. Effect of random eccentricity on the shape of the projection of the load-bearing capacity area of a reinforced concrete section

The origin of such inconsistencies is related to the fact that the traditional approach based on the manual calculation generated all sorts of "simplifications", which allowed to skip some checks or to replace the general case with a certain particular case (as at $m < 20$ for steel elements under bending and compression).

Moreover, the use of “logical switches” that change rules without an exact physical basis leads to an abrupt change in the algorithm, as in the classification of cross-sections in the EuroCode. Modern technologies enable to detect such inaccuracies and determine ways to improve the codes.

LOAD-BEARING CAPACITY AREA OF JOINTS AS A TOOL FOR ANALYZING TRADITIONAL APPROACHES TO THEIR CALCULATION

The design and calculation of the joints of bar structures is, apparently, one of the most important stages in the design of steel structures. Unlike the analysis of the stress-strain state based of a structural design model, which obeys the strict rules of structural mechanics, the “algorithms” for calculating joints use traditional (taking into account the previous experience) methods of approximate solution, which are based on simplified notions about the operation of joints. Thus, the design models of joints of steel structures are largely determined by the existing design traditions. As a rule, these techniques are closely related to a set of service-tested designs of joints used for this type of structures.

Similarly to the definition of the load-bearing capacity area of a section introduced earlier, we introduce the definition of the load-bearing capacity area of a joint.

A *load-bearing capacity area of a joint* $\mathbf{\Omega}_\mathbf{J}$ is a certain area in the *k*-dimensional space of internal forces, all points of which correspond to a combination of these internal forces, under the action of which the joint satisfies *all the standard requirements*. The boundary of the load-bearing capacity area of the joint is the entire set of limitations governed by the requirements given in the codes (including the strength conditions of the sections of the load-bearing structural elements adjacent to the considered joint, the strength conditions for the structural elements of the joint – wing plates, cantilever stiffeners, base plates and end-plates, the strength conditions of bolted and welded connections in a joint, etc.). The dimension k of the load-bearing capacity area of the joint is defined by the number of load-bearing structural elements l adjacent to the considered joint, as well as by the total number n of internal forces that may occur in the sections of these elements, $k = l \times n$.

COMET (Karpilovsky *et al.* 2014) enables to perform the computer-aided generation of the load-bearing capacity areas of the joints of steel structures in terms of the considered codes for rigid and nominally pinned column bases, beam splices, beam-to-column joints, and for truss panel points.

When the calculation of the joints of steel structures is performed according to the codes, the considerations and hypotheses that were developed during the times of manual calculation and served as a means of simplifying the manual calculations of engineers are used here. “Traditional” models are often very coarse, because they were developed during the “manual” design period, which required maximum simplification of the problem. The use of these conservative simplifications in the computer analysis often indicates the inconsistency of individual hypotheses.

The shape of the load-bearing capacity area of the joints, as well as the character of its boundaries, has made it possible in many cases to analyze the traditional approaches to the calculation of the joints of steel structures in more detail. The analysis of the boundaries of the area has shown that the simplifications of the formulations of the regulations of the type “can be ignored” (as, for example, an indication of the possibility of neglecting the end-plate stiffeners when determining the geometric properties of the support section of the girder and the distribution of normal stresses in it, which is given in (Recommendations for the Calculation, Design, Manufacture and Installation of End-plate Joints of Steel Structures 1989) or “can be taken” (as, for example, the forces in anchor bolts can be taken as equal to their load-bearing capacity or averaged over the sections of uniform distribution of the reaction of the foundation concrete under the base plate), in most cases lead to various kinds of non-convexities in the load-bearing capacity area of the joints.

Therefore, the software implementation of the calculation of the load-bearing capacity of joints of steel structures required the developers to avoid any simplifications of regulations. And in some cases, the refusal to simplify the formulations of the regulations was inevitable. For example, the traditional methods of calculating the joints of steel structures given in the codes are not always oriented toward solving three-dimensional problems. In particular, this refers to the rigid column bases, for which the problem of determining the contact area of the base plate with the foundation concrete arises under bending in both principal planes. In the case when there are commensurate bending moments in the support section of the column and, respectively, tensile forces arise in the anchor bolts, the stress distribution in the foundation concrete, as well as the forces in the anchor bolts, are calculated in **COMET** by the finite element method using an equivalent reinforced concrete section with the dimensions equal to those of the base plate and the arrangement of the working reinforcement, corresponding to the arrangement of anchor bolts in the base (Fig. 8). For such an equivalent section, the stresses at the characteristic points below the base plate are determined, in particular, depending on the length of the perpendicular dropped to the neutral axis of the section.

Thus, the massive transition to CAD-CAE design systems enables to use more accurate design models of steel structural joints in everyday practice. However, the results of this “refinement”, often leading to a complication of the joint, may require additional justifications.

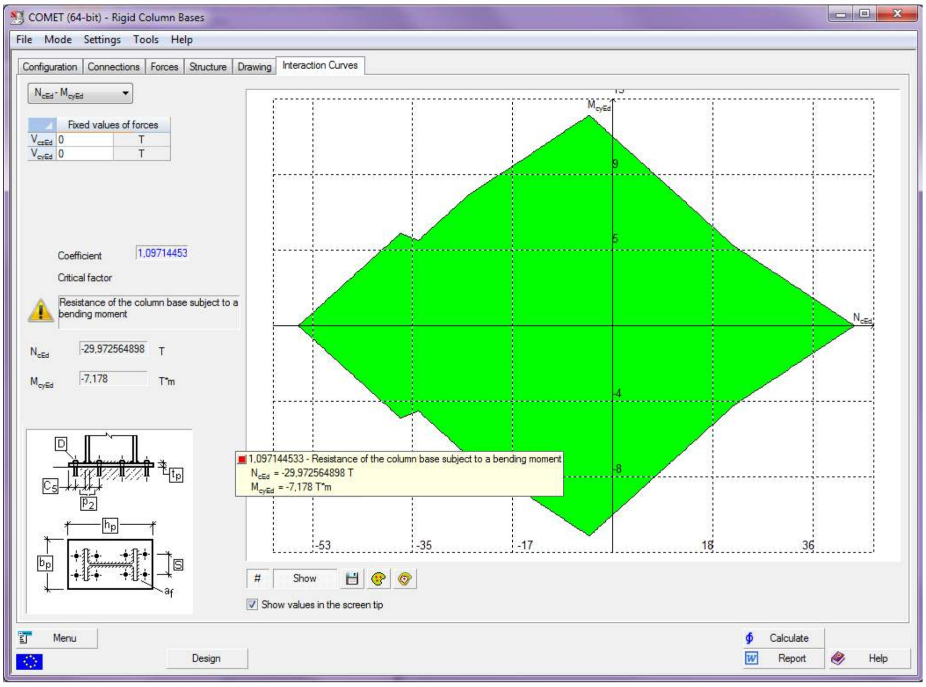

Figure 8. The load-bearing capacity area of the rigid column base using an equivalent reinforced concrete section when calculating the stress distribution in the foundation concrete and the forces in the anchor bolts

LOAD-BEARING CAPACITY AREA OF SECTIONS AS A TOOL FOR ANALYZING LOADING CONDITIONS

Dangers related to the non-convexity of the load-bearing capacity area seem to deprive us of the grounds for using the methods given in the codes. However, the vast experience in using the recommendations of these documents speaks in favor of the fact that these dangers do not realize. Apparently, the real variants of loading turned out to be such that the system avoided the “dangerous zone”. This fact points to the possibility of analyzing the closeness of the real set of internal forces to those boundary of the load-bearing capacity area, where the property of non-convexity is manifested.

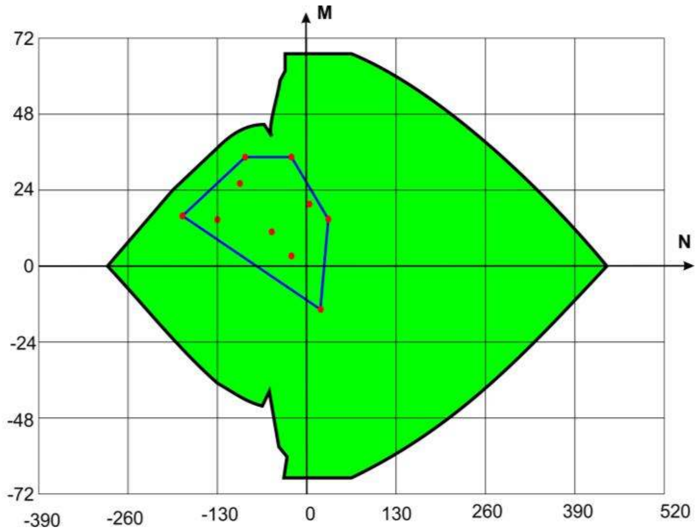

Figure 9 Given (basic) loadings and their convex hull, combined with the load-bearing capacity area of the section

An almost identical analysis can be performed using additional tools provided by SCAD Office components. These tools enable to display the entire set of verified loading options (combinations of internal forces specified by the user), in the form of a set of points, each of which corresponds to one of the loading options (Fig. 9). A convex hull of these points is also shown, i.e. a set of loadings that are linear combinations of the given (basic) loadings. Despite the fact that these combinations were not subjected to a direct check, in the case when the convex hull of loadings does not leave the load-bearing capacity area of the section, it can be ensured that the various loadings combined from the basic ones are not dangerous.

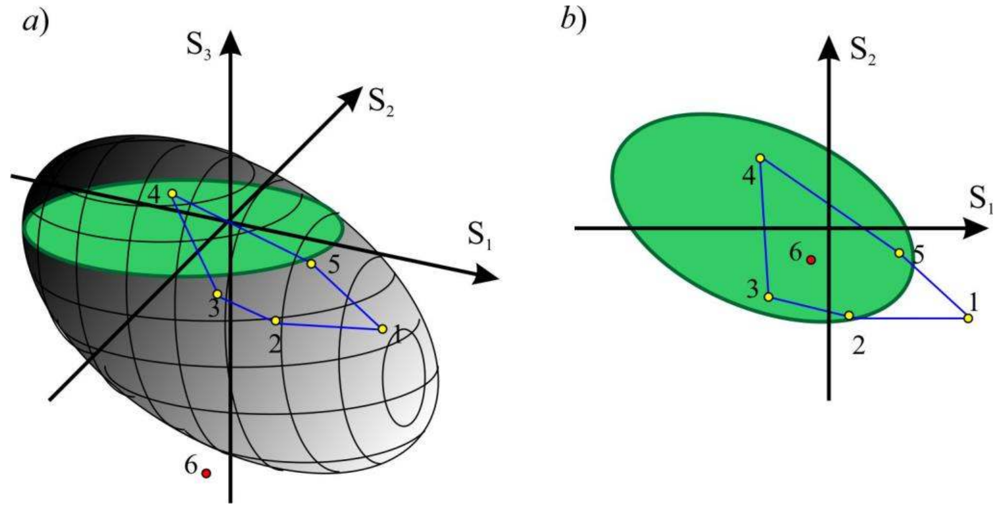

Figure 10. Illustration of possible design situations

It should be noted that when operating only with two-dimensional orthogonal projections of the load-bearing capacity area, it is possible to “see” the projections of some points (combinations of internal forces) belonging to this area (for which the maximum utilization factor of restrictions does not exceed one) as displayed outside the projection boundary of the load-bearing capacity area (as, for example, point 1 in Fig. 10). There may also be an erroneous “vision” of a different kind, when the projection of the point lies within the boundaries of the projection of the load-bearing capacity area, and the point itself does not belong to the area (see, for example, point 6 in Fig. 10). In order to identify such situations, the projections of the points in which the utilization factor of restrictions exceeds one, are displayed in red on the projections of the load-bearing capacity area.

However, we can not be completely sure that the used set of basic loadings is accurate, since all our calculations operate with the initial data determined only with a certain accuracy. We can only assume that the real set of possible variants is not very far from the convex hull of the base loadings. Thus, the complete immersion of the convex hull of the base loadings in the load-bearing capacity area is only a necessary condition, and a more cautious recommendation is that the convex hull of the base loadings should be located at a sufficient distance from the non-convex section of the boundary.
As a measure that defines the concept of "sufficient distance" we can suggest the following procedure. We can determine the modulus of the internal force vector for each base point

$$|S| = \sqrt{\left(S_1/S_1^0\right)^2 + \left(S_2/S_2^0\right)^2 + \ldots + \left(S_n/S_n^0\right)^2} \, ,$$

where S_i^0 denotes the value of the i^{th} force at the boundary of the load-bearing capacity area. Forces S_i were calculated on the basis of a set of initial data on the loads on a structure, which, generally speaking, are approximate, since it is hardly possible to take into account all the circumstances of structural loading without exception and with an exact precision.
Assuming that the initial data are determined with the usual for engineering practice accuracy of up to 5%, we can say that the convex hull of the base loadings is at a sufficient distance from the dangerous boundary if it does not approach it by a distance less than $0,05|S|$.

CONCLUSIONS

Load-bearing capacity region for structural sections and joints in terms of design codes has been considered. The main attention has been paid on important properties of the region.
Algorithm for automatic generation of load-bearing capacity regions for structural sections, connections and joints has been presented and implemented in SCAD Office software package.
Construction of load-bearing capacity regions of a section or joint combined with a convex shell of specified internal forces is flexile tool for analysis of load conditions. Load-bearing capacity region for cross-sections is also a mean for critical analysis of code-based requirements and regulations used for verifications of cross-sections. Finally, load-bearing capacity regions has been also used in process of software development as an instrument for critical analysis of traditional approaches to design and calculation of steel structural joints.

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