# STABILITY OF A STEEL TUBE ARCH SUPPORTING BARREL TYPE TEXTILE ROOF MEMBRANES

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## ABSTRACT

Following extensive tests of a single (alone) built-in tubular steel arch and the identical arch supporting textile membrane roofing fastened to the arch, the corresponding numerical analyses are presented. The membranes made of the prestressed PVC coated polyester fabric Ferrari® Précontraint 702S were used as a currently standard and perfect material. The tests and results oriented to the loss of in-plane and out-of-plane stability of the arch are briefly described. SOFiSTiK software was employed to model orthotropic membrane and supporting steelwork behavior using geometrically nonlinear 3D analysis with imperfections (GNIA). The numerical results were validated using test data based on various membrane prestressing with excellent agreement. Finally results of vast parametric studies concerning stability of a central arch in five arches membrane assemblies are presented. Enormous savings in the tubular steel arches due to the membrane supporting effects are revealed, leading to omission of a stability check concerning the out-of-plane arch buckling.

## INTRODUCTION

Tensioned Fabric Structures (TSF) are becoming more and more popular for their visual attractiveness, lightness and sufficient load-bearing capacity. Currently various membrane materials are available for both single layer double curved shapes (barrels, hypars, cones) and double/more layer pneumatic constructions (inflatable cushions, air supported pneumatic structures). The PVC coated polyester fabric (e.g. Précontraint FERRARI®) seems to be a rational choice for common single layer structures with the lifetime up to 20 years, acceptable cost and good joining possibilities (welding or sewing). Details on other materials (as glass fabrics, expanded PTFE, polyethylene fabrics, ETFE, THV) are described e.g. by Machacek and Jermoljev 2016.

Design of structures with TSF requires geometrically and materially non-linear analysis with imperfections (GMNIA). In the analysis an appropriate input of the membrane material behaviour is required. The material is due to its structure non-homogeneous, orthotropic (warp and fill directions) and non-linear. The most sophisticated model with 15 input parameters obtained from laboratory tests was proposed by Kato *et al.* 1999. The model demonstrates excellent agreement with test data but because of its complexity and time requirements is rather unsuitable for practical utilization. Apart from a simplified approach, Gosling 2007 suggested "strain-strain-stress" approach using response surfaces linking strains to stresses through three dimensional representations. A non-linear material model with 5 parameters based on experimental results and depending on load ratios in warp and fill direction was proposed by Galliot and Luchsinger 2009. Model parameters are: warp and fill Young's modules for the respective load ratio 1:1, the change in warp and fill Young's modules (variation of the moduli on the whole range of load ratios) and the Poisson's ratio. A simplified stress-strain model for coated plain-weave fabrics was developed by Pargana and Leitao 2015. Model consists of three nonlinear elements to model the yarns and an isotropic plate to model the coating.

Nevertheless, in accord with recommendation of Tensinet Analysis & Materials working Group (Foster and Nollaert 2004) a simplified elastic approach may be employed using the simple plane stress theory. The supplied test data provides elastic modules for warp and fill directions and corresponding Poisson's ratio, valid for anticlastic type of structures. Recently Uhlemann *et al.* 2015 analysed two approaches concerning simplified elastic constants for design of the membranes (Japanese MSAJ and acc. to European Tensinet & Materials working Group) and found the European approach more general and reasonable for PES/PVC material, but still with reservations.

For novel unique structures the use of supporting components as slender as possible is necessary to follow the concept of a delicate, light and attractive structure with the membrane surface and to avoid any visual intervention of supporting steelworks into the membrane area. While membrane surface is exclusively tensioned, supporting construction is most often exposed to a compression and/or bending. This type of loading, in combination with slender elements, results into stability problems and design must be done with respect to these effects. Different situations occur, when membrane surface is joined with the supporting steel structure continually. In this case the membrane represents a spring support for the supporting structure, the critical (buckling) length of individual steel elements is changing, and both parts of the entire system cannot be investigated separately but as one complex structure using proper software package which allows integrated modelling and computing (e.g. EASY (technet), FORTEN (ixForten), SOFiSTiK, Rhino Membrane, NDN (membrane NDN)], etc.). Detailing of membrane structures and erection methods are well described by Seidel 2009, and the basic design in European Design Guide 2004.

The paper deals with stabilizing effects of membranes to the respective supporting steelwork, based on numerical parametrical studies validated by tests. The tests relate to the structural model representing a concert stage structure with two supporting arches. The stability of the inner supporting arch is of the primary interest. Influence of various membranes prestressing on the structure stabilization is analysed and compared with the test results. Parametrical study concerns nonlinear behaviour of tube arches with various geometries which support textile membranes.

## LABORATORY TESTING

### Model Arrangement

Model of an outdoor covered stage (with an approx. reduction 1:10) was proposed using Formfinder Software. The software enables intuitive manipulation with membrane shapes under required stress level in interactive way and export/import to other programs through DXF/DWG files. Supporting steelwork involves two supporting arches (outer and inner CHS tubes), bottom edge steel wire ropes and membrane plates, see Figure 1.

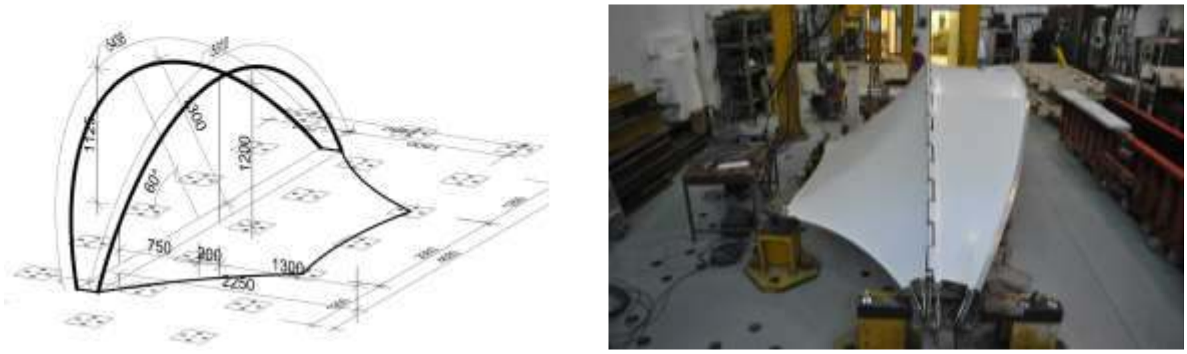


Figure 1: Setup and photo in laboratory.

The membrane is PVC coated polyester fabric Ferrari® Précontraint 702S with opaque surface (weight 830 g/m$^2$). Principal dimensions of the vertical inner tubular arch are $L$x$H$ = 4500x1200 [mm], while outer tubular arch has inclination of 60° in respect to horizontal.

The fabric of Serge Ferrari group is made of polyester scrim coated both sides with liquid PVC and PVDF topcoat, weldable for joining. The material is due to its structure non-homogeneous, orthotropic (warp and fill directions) and non-linear, with elastic constants presented further at chapter of numerical modelling. Nevertheless, the patented fabrication process ensures similar elongation behaviour in both warp and fill directions and minimum creep. Concerning material characteristics the biaxial test performed by Lab BLUM Stuttgart 2005 is available. With respect to these tests both warp and fill braking loadings were considered as $S_{ult} \approx 56$ kN/m, while working loading $S_{max} = S_{ult}/5 \approx 11.2$ kN/m to exclude tearing and prestressing up to $P = S_{max}/5 \approx 2.24$ kN/m. Nevertheless, prestressing in the case of the lab model was up to $P \leq 0.5$ kN/m only.

The steel tubes from grade S355J0 were hot-formed in workshop. The investigated inner tube of ø 26.9x3.2 [mm] and outer one of ø 88.9x3.2 [mm] were welded to steel blocks to form a fixed-in frame acc. to Figure 1. Coupon tests of the inner steel tube resulted into average yield strength $f_y$ = 475 MPa, ultimate strength $f_u$ = 595 MPa and elongation 27.1 %. Modulus of elasticity was considered in accord with Eurocode 3 as $E$ = 210 GPa.

The membrane was joined with the outer arch using riveted aluminium keder profile while to inner tube via alternating pockets. Common 7x7 wire peripheral rope from CarlStahl Company with diameter of 6 mm in curved cuff fastened the membrane to corner plates and anchors. The investigated inner arch was fitted with transducers (electrical potentiometers) in vertical (V), transverse (H) and longitudinal (L) directions to measure deflections. Their positions and loading points (P) are shown in Figure 2. In supports and middle of the arch strain gauges denoted (T) were placed, always in four mutually perpendicular positions, for later comparison of stresses with numerical results.

The prestressing of the membrane resulted from the membrane cut as prepared by EASY (technet) software, considering uniform prestressing of roughly $P$ = 0.5 kN/m. During assembly the peripheral ropes were tightened and the membrane checked against wrinkling. Eight strain gauges were placed at various locations within the membrane surface for rough information on prestressing values. The measuring before and after prestressing resulted into average prestrain value of $\varepsilon_y$ = 140 μm/m (i.e. $P \approx$ 0.09 kN/m) in perpendicular direction with respect to the supporting arches and $\varepsilon_x$ = 450 μm/m (i.e. $P \approx$ 0.30 kN/m) in parallel direction to the arches.

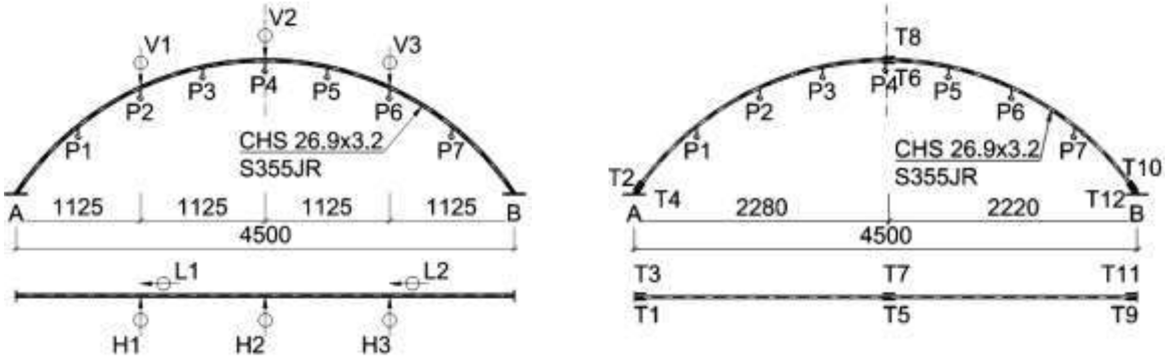


Figure 2: Positions of loading (P), transducers (V, H, L) and strain gauges (T).

## Test Results

The investigation concerned exclusively the inner steel arch to find stabilizing effect of the membrane to its nonlinear behaviour. First the inner arch alone (without fastening the membrane) was loaded and second, after the membrane assembly, the complete membrane structure. For loading calibrated pouches with steel pellets were used and suspended from seven given points $P$ at the arch. The loadings were carefully arranged to simulate uniform symmetrical loading and asymmetrical loading corresponding to the first in-plane buckling mode. Maximum loadings for arch alone are given in Figure 3, for complete membrane structure in Figure 4.

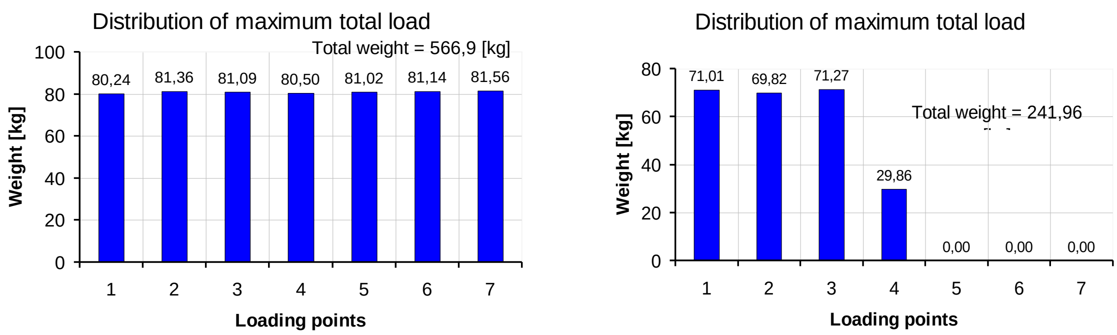


Figure 3: Loading of the inner steel arch alone:
Left-symmetrical, right - asymmetrical.

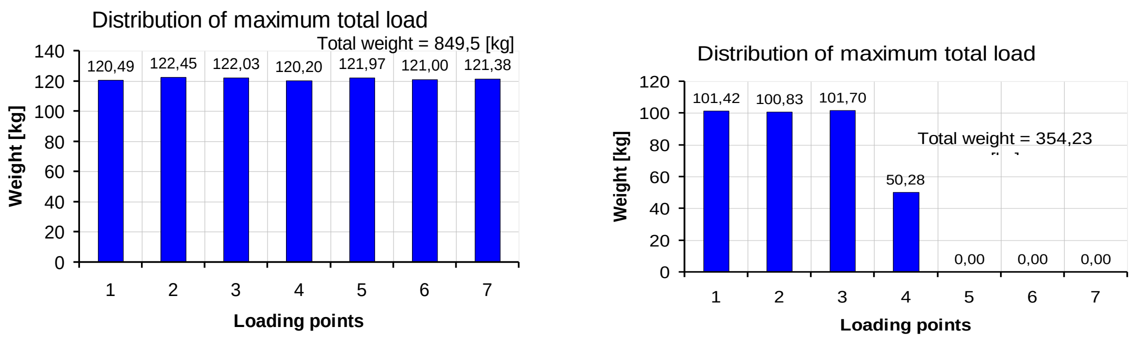


Figure 4: Loading of the arch joined with the membranes:
Left-symmetrical, right – asymmetrical.

Individual loading steps amounted for roughly 1/10 of maximum loading, each followed by unloading. The tests were terminated when abnormal deflections out-of-arch plane or in-arch-plane were reached.

## Results for Symmetrical Loading

Deflections of the inner arch under increasing loading in vertical and horizontal directions are shown in Figure 5 (unloading is not included and generally was fully elastic). The arch without membrane buckled out-of-plane at total loading approaching $F_0$ = 5.5 kN, with associated vertical deflection along all span down. On the other side the test with arch stabilized by the membrane was terminated under total load of $F_M$ = 8.3 kN, showing very small and nearly linear increase of the mid span deflection. Stabilizing effect of the membrane is enormous.

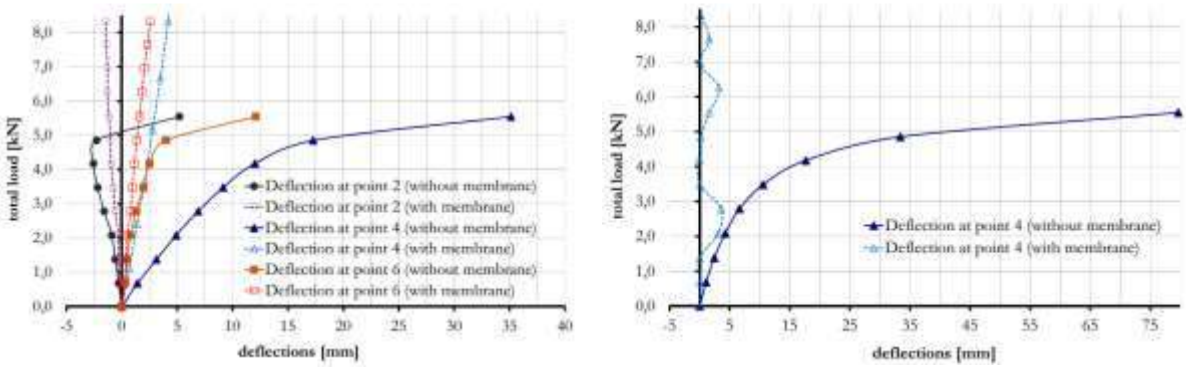


Figure 5: Symmetrical loadings:
Vertical deflections (left), transverse deflections (right).

## Results for Asymmetrical Loading

Vertical and transverse horizontal deflections under increasing loading are shown in Figure 6. Testing of the arch without membrane ("0") terminated under total loading $F_0$ = 2.37 kN, giving maximal vertical deflection $\delta_0$ = 41.3 mm and horizontal one $\eta_0$ = 3.5 mm. The arch stabilized by membrane ("M") deflected much less, giving for the same loading $F_M$ = $F_0$ = 2.37 kN values $\delta_M$ = 18.5 mm and $\eta_M$ = 0.5 mm. The stabilizing effect of the membrane concerning vertical deflection resulted into reduction of 45 %.Difference comparing vertical deflections in positions V1 and V3 (see Figure 2) gives $\Delta\delta_0$ = 68.3 mm and $\Delta\delta_M$ = 32.9 mm.

Measured displacements and stresses (not presented) prove expected enormous effect of the membrane on buckling load and strength of inner supporting arch, particularly in asymmetrical loadings. Test values are used for validation of numerical analyses.

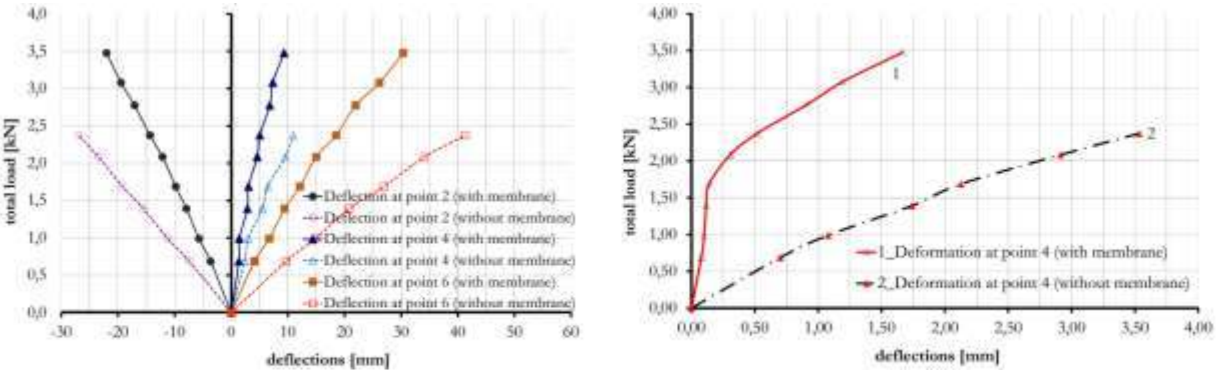


Figure 6: Asymmetrical loadings:
Vertical deflections (left, positive down), transverse deflections (right).

## NUMERICAL ANALYSIS

### Numerical Model and Validation by Tests

SOFiSTiK software 2014 was used to perform GNIA (using N-R iteration) both for the inner arch alone and the complete membrane structure. Various meshing of the membrane was analysed (square sizes of 25, 50, 100 and 200 [mm]) giving nearly identical stress results (differences ≤ 0.2%), but with increase of computer time between largest and least mesh up to 107 times. Therefore, optimum mesh with 50 mm size was used throughout the analysis.

### Arch Alone Analysis

The tube arch was introduced as ø 26.9x3.2 [mm], with built-in supports, material S355J0 and Young's modulus $E$ = 210 GPa. Initial geometry as measured was introduced into analysis (theoretical radius $R$ = 2709 mm with out-of-plane (transverse) deflection at the crown $w_0$ = 17 mm). The elastic buckling analysis of the perfect arch under symmetrical loading in accordance with Figure 3 (left) resulted into total critical loads $TL_{cr,1}$ = 6.7 kN (out-of-plane single wave buckling), $TL_{cr,2}$ = 15.4 kN (out-of-plane two waves buckling), $TL_{cr,3}$ = 18.7 kN (in-plane two waves buckling), etc. GNIA transverse deflection curves for symmetrical loading are shown in Figure 7 left, indicating the first out-of-plane bifurcation earlier but in accordance with test, roughly at 4.9 kN. Vertical deflections for asymmetrical loading correspond also well to test results, see Figure 7 right.

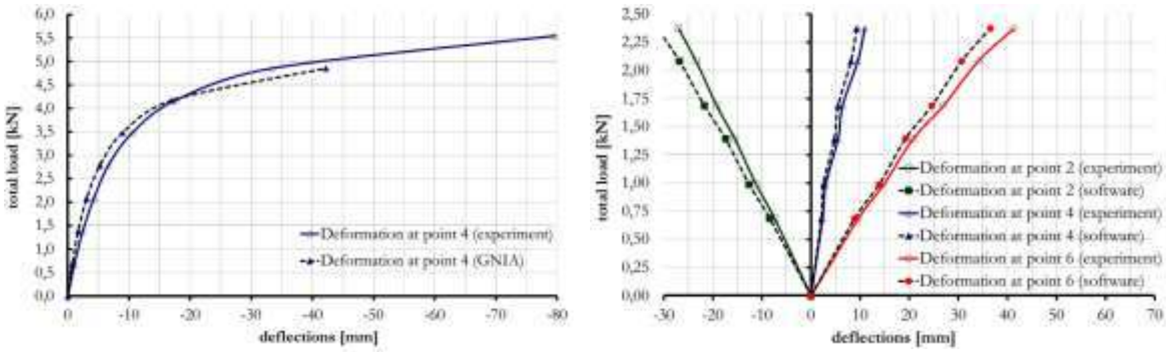


Figure 7: Arch alone, test and GNIA results.
Left: transverse deflections for symmetrical loading at midspan.
Right: vertical deflections for asymmetrical loading.

**Full Membrane Structure Analysis**

The membrane FERRARI® 702 was considered with linear elastic orthotropic behaviour taking account for influence of load ratios according to Galliot and Luchsinger 2009. The necessary five parameters were taken as: warp and fill Young's moduli for 1:1 load ratio $E_w^{1:1}$ = 635.3 kN/m and $E_{wf}^{1:1}$ = 661.9 kN/m, the change in warp and fill Young's moduli $\Delta E_w$ = 295 kN/m and $\Delta E_f$ = 168 kN/m, the Poisson's ratio $v$ = 0.196. The warp direction was considered as perpendicular to the arch plane and conversion to stresses due to approximate thickness of the membrane as 0.7 mm.

Initial geometrical imperfections of the arch were measured just before the first loading and were found to be 5.0 mm in horizontal direction at the top of the arch and +/-10.0 mm in vertical direction at the quarters of the arch. Initial shape of the membrane surface was generated automatically by the AutoCAD as a surface with minimal area. The shape was exported into SOFiSTiK software. For each level of the membrane prestress the corresponding value of the perimeter cable pretension was calculated based on formula:

$$S_S = r \cdot S_M \qquad (1)$$

where $S_S$ is a tension force in a perimeter cable [kN],
$r$ radius of the cable curvature [m],
$S_M$ prestress in the membrane perpendicular to the cable [kN/m].

The equilibrium state and final unloaded shape of the structure was found under initial SOFiSTiK software calculations. The results of GNIA under various pretension of the membrane $P$ [kN/m] is shown in Figure 8. The level of pretension is the crucial parameter for the arch stability and resistance capacity. Higher pretension in membrane logically means higher stability and smaller deflection of the investigated arch. It should be recalled, that the real pretension during the testing was influenced by the alternating "pockets" connection of the membrane to the inner arch and due to this fact rather non-uniform and slacked. As mentioned, the randomly measured values during the test at several localities (which however can't be considered as a reliable ones), were between 0.09 and 0.3 [kN/m]. This is reflected in numerical analysis with pretension close to zero (0.2 kN/m) which provides results very near to the test ones. Therefore, the received GNIA results with the corresponding prestress 0.2 kN/m justify use of the model for following parametric studies.

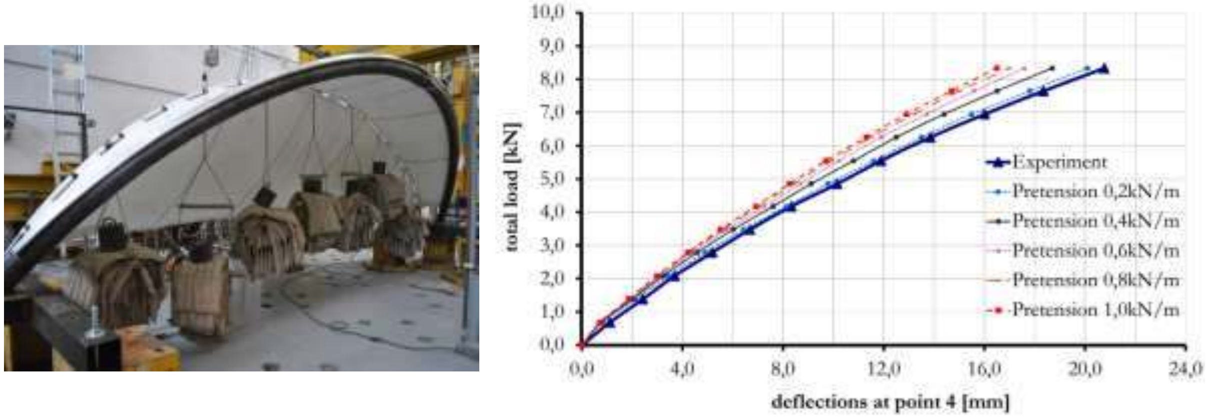


Figure 8: Symmetrical loading of the membrane model:
Comparison of the test and GNIA vertical deflections under various pretension.

**Parametrical Study**

The principal goal of the study was to determine the stabilizing effect of common textile membranes on buckling and nonlinear behaviour of slender steel supporting arches. Both in-plane and out-of-plane arch buckling was studied for various geometries of arches and membranes in a parametrical study. Inner arches in the 5 arch assembly with practical dimensions arranged in a barrel type structure in according with Figure 9 were investigated. To evaluate the influence of lateral boundary conditions, the outer (edge) arches were either flexible (supported by a truss) or continuously fixed in all three directions.

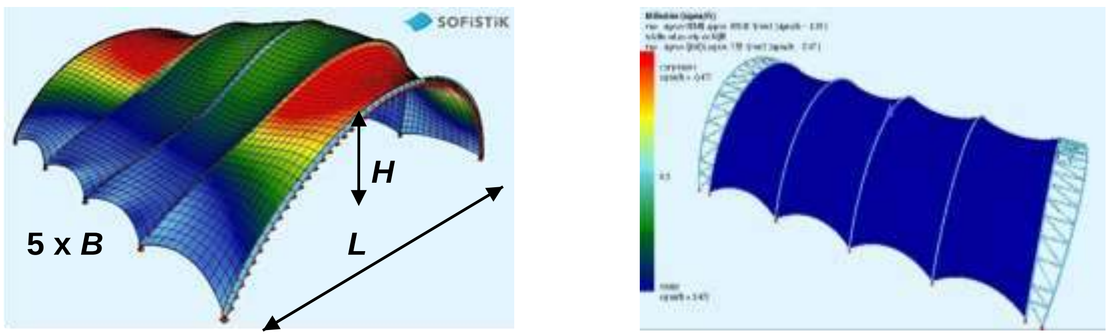


Figure 9: Five arch assembly and outer lateral flexible support.

Parametrical study covers the arch span of $L$ = 6 ÷ 20 [m], rise $H$ = $L$/10 ÷ $L$/2 and spacing of the five arches $B$ = 3 ÷ 10 [m]. Loading was considered in vertical direction (alternatively in radial direction) as 1 kN/m$^2$ simulating snow loading only, together with various prestressing of the membranes, ranging from 4 to 7 kN/m in both directions. The arch dimensions were designed for resulting internal forces according to Eurocode 3 from steel grade S355 in an iterative way, to reach 80÷90 % of their design capacity.

First the buckling of the arch alone with built-in supports was investigated to find in-plane and out-of-plane buckling loads under considered loading (Figure 10).

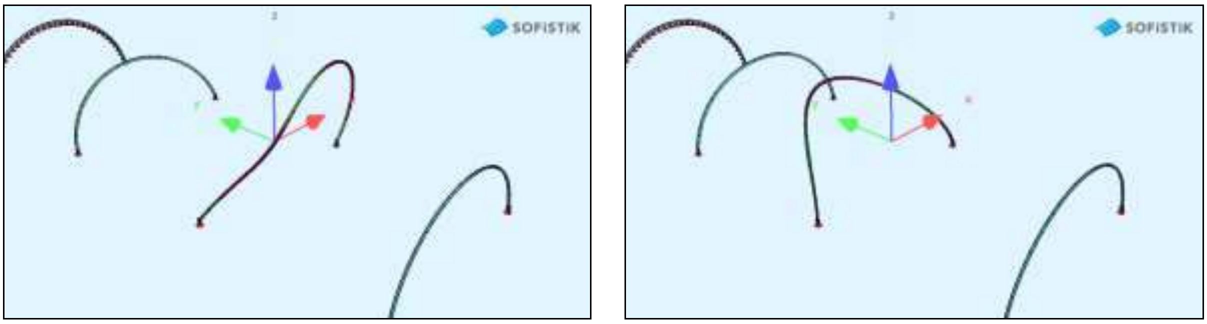

Figure 10: Arch alone: in-plane buckling (left), out-of-plane buckling (right).

Second the all five arch assembly (including lateral support) was analysed to find in-plane and out-of-plane buckling loads for the mid-arch under loading including various prestressing of the membranes (Figure 11).

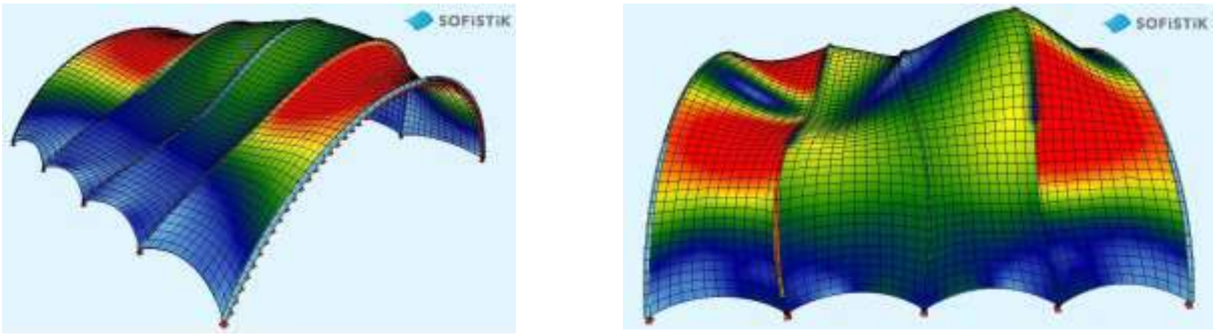

Figure 11: Arch with membranes: mid arch in-plane buckling (left), mid arch out-of-plane buckling (right).

For example tube arch ø 133x5 [mm], steel grade S355, with $L$ = 12 m, $H$ = 3.6 m and $B$ = 3 m and uniformly loaded vertically under 3 kN/m with mesh size 250 mm, gives following buckling loads: arch alone in-plane buckling $N_{cr.y}$ = 189 kN, out-of-plane buckling $N_{cr.z}$ = 70 kN; arch with membranes under vertical loading 1kN/m$^2$ and prestressed by $P \approx 5.0$ kN/m gives $N_{cr.y}$ = 346 kN and $N_{cr.z}$ = 542 kN. Similar relations were obtained throughout the parametrical study, indicating that the membrane support to the stability of arch is substantial.

## Conclusions

The main conclusions are:

1. Tests confirmed decisive effect of textile membranes on both in-plane and out-of-plane supporting arch stability and strength.
2. GNIA with linear elastic orthotropic behaviour of the textile membrane and using SOFiSTiK software proved to be adequate for numerical modelling.
3. The real value of the membrane prestressing substantially influence the deflections and strength of the supporting arch structure (see the sensitivity study in Figure 8).

4. Parametric studies of large barrel membrane structures supported by a row of steel arches resulted into huge savings in a practical design of supporting steel arches due to enormous increase of both in-plane and particularly out-of-plane buckling loads in comparison to the ones of an arch alone (e.g. in the shown case $N_{cr.z}$ increased 7.7 times).
5. Within the scope of the parametric study was shown, that in cases of a tube arch alone the in-plane buckling of the arch is not decisive ($N_{\mathrm{cr.in}} \approx 2 \div 4\ N_{\mathrm{cr.out}}$), while for the arch with membranes the out-of-plane arch buckling is not decisive ($N_{\mathrm{cr.out}} \approx 2 \div 3\ N_{\mathrm{cr.in}}$). Therefore, for tube arches supporting membranes the simplified design considering the in-plane buckling only is sufficient.


## ACKNOWLEDGEMENT

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